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Isoscalar off-shell effects in threshold pion production from pd collision

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We test the presence of pion-nucleon isoscalar off-shell effects in the $pd \to \pi^+ t$ reaction around the threshold region. We find that these effects significantly modify the production cross section and that they may provide the missing strength needed to reproduce the data at threshold.

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In the past decade, with the aim to clarify the nature of the elementary $NN\to\ \pi NN$ s-wave inelasticities, a great deal of experimental and theoretical activity has been made in pion production from NN collision at energies close to threshold. The situation has been recently reviewed by H. O. Meyer [1]. Advances in experimental techniques allowed to measure in particular the reaction $pp \to \pi^o pp$ cross section very close to threshold. This reaction filters the s-wave πN coupling in the isoscalar channel. Standard theory including the one-body term and isoscalar rescattering constrained by the πN scattering lengths underestimated the cross section by a factor of five. Unexpectedly, there have been two theoretical explanations for this discrepancy, not just one. The enhancement in the cross section can be explained by invoking short-range nucleon-nucleon effects [2], where the important effects can be simulated by σ and ω exchanges [3]. The explanation is appealing, since in this case the pion field couples with the two-nucleon axial charge operators, and this coupling provides an explicit link to the inner part of the nucleon-nucleon interaction, which is notoriously difficult to disclose. But the results have been entirely explained by resorting also to an off-shell enhancement of the isoscalar πN amplitude [4]. The original calculation by Hernández and Oset with a σ-exchange model for πN scattering was subsequently confirmed with a more microscopic πN model, with the finding that the off-shell increase is substantially confirmed, but may be significantly reduced in size in passing from the original Hamilton model [5] to a phenomenological one [6].

The three-nucleon system is a richer testing ground for studies of pion production and scattering. The addition of just one nucleon increases the complexity of the reaction which involves now the simplest nontrivial multinucleon system where it is possible to test the fundamental $NN\to\ \pi NN$ process and, at the same time, to solve accurately the nuclear dynamics with Faddeev methods. Applications of Faddeev methods to pion production/absorption started very recently with studies centered around the Δ resonance [7,8] and herein we apply the same technique of Ref. [8] to study pion production at threshold. Besides the obvious difficulty of performing calculations with three nucleons instead of two, one encounters here the additional difficulty that for the $pd \to \pi^+ t$ reaction one must include from the start the effect of p-wave mechanisms, on top of the s-wave ones. This contrasts with the findings for the two-nucleon case, where the effect of the p-wave mechanisms (including the Δ), tends gradually to zero in approaching the threshold limit. The effect of this difference can be immediately perceived in the differential production cross section, since for NN collision the angular dependence evolves gradually with energy, while in the case of pd collisions it exhibits a remarkable s- and p-wave interference in the threshold region, with strong forward-backward asymmetry [9].

In this letter, we have centered our study on the effects due to the s-wave rescattering processes, taking into account both isoscalar and isovector components and their interference to the p-wave mechanism (containing the Δ degrees of freedom). We have in particular taken into account the off-shell effects in the isoscalar πN coupling by following the same prescription suggested in Ref. [4] to explain the size of the excitation function for the $pp \to pp\pi^o$ process. Since the importance of the isoscalar off-shell effects in pion production from NN collisions has been there established and later confirmed with an independent calculation, it is clearly of great relevance to consider the consequences of such effects on more complex reactions. Herein we provide the results obtained when calculating such effects in pd collisions at threshold.

The production mechanisms are constructed starting from the following effective pion-nucleon couplings

$$\mathcal{L}_{\rm int} = \frac{f_{\pi NN}}{m_\pi}\bar{\Psi}\gamma^\mu\gamma^5\vec{\tau}\Psi\cdot\partial_\mu\vec{\Phi} -4\pi\frac{\lambda_I}{m_\pi^2}\bar{\Psi}\gamma^\mu\vec{\tau}\Psi\cdot\left[\vec{\Phi}\times\partial_\mu\vec{\Phi}\right] - 4\pi\frac{\lambda_O}{m_\pi}\bar{\Psi}\Psi\left[\vec{\Phi}\cdot\vec{\Phi}\right]. \tag{1}$$

The first term represents the gradient coupling to the isotopic axial current, while the second denotes the coupling to the isovector nucleonic current, and the last is the pion-nucleon coupling in the isoscalar channel.

As well known [10], a good guess for the coupling constants can be obtained from chiral symmetry and PCAC, which constrain the three constants to be of the order

$$f_{\pi NN}/m_\pi \simeq g_A/(2f_\pi) \tag{2}$$

$$4\pi\lambda_I/m_\pi^2 \simeq 1/(4f_\pi^2) \qquad 4\pi\lambda_O/m_\pi \simeq 0, \tag{3}$$

where g_A ($\simeq$ 1.255) is the axial nucleonic normalization, and f_π is the pion decay constant ($\simeq$ 93.2MeV). The first condition follows directly from the Goldberger-Trieman relation, while the last two are implied by the Weinberg-Tomozawa ones. Current phenomenological values for $f_{\pi NN}^2/(4\pi)$ can reach values as low as 0.0735 [11], or 0.0749 [12], but also values around 0.081 [13] have been considered acceptable. Some years ago common values were centered around 0.078–0.079 [14]. Similarly, from the pion-nucleon scattering lengths λ_I is determined within the range 0.055–0.045, while the weaker isoscalar coupling has typically larger indetermination, ranging from 0.007 to -0.0013 [2,4]. The isovector and isoscalar couplings, when combined with the axial πNN vertex, are the basic ingredients for the two-nucleon s-wave rescattering mechanisms, while the axial-current coupling alone forms the basis for the one-body production process.

The matrix elements for the rescattering process require an off-mass-shell extrapolation of the two constants λ_I and λ_O, since the rescattered pion can travel off-mass-shell. For λ_O we consider the off-shell structure previously employed in the $pp \to pp\pi^o$ process by Hernández and Oset (Ref. [4]) which is based on a parameterization due to Hamilton [5], namely

$$\begin{aligned}\lambda_O(\tilde{q},\tilde{p}) &= \lambda_O^{\rm on} g_O(\tilde{q},\tilde{p}) \qquad (4)\\ &= -\frac{1}{2}(1+\epsilon)\, m_\pi \left(a_{sr} + a_\sigma \frac{m_\sigma^2}{m_\sigma^2-(\tilde{q}-\tilde{p})^2}\right),\end{aligned}$$

with $m_\sigma = 550$ MeV, $a_\sigma = 0.22\, m_\pi^{-1}$, $a_{sr} = -0.233\, m_\pi^{-1}$, and $\epsilon = m_\pi/M$. In the threshold limit, $(\tilde{q}-\tilde{p})^2 \simeq (\tilde{q}_o - m_\pi)^2 - \tilde{\mathbf{q}}^2$, where $\tilde{q}$ is the transferred 4-momentum between the two active nucleons. According to previous treatments of the $2N$ π-production amplitude, the time-component $\tilde{q}_o$ is fixed around $\tilde{q}_o \simeq m_\pi/2$, while the space component $\tilde{\mathbf{q}}$ represents a loop variable and is integrated over.

This form leads to an on-shell value of the order of 0.007. The on-shell value derives from a cancellation between the short-range term, a_{sr}, and the σ-exchange term, a_σ, while off shell the cancellation occurs only partially and thus leads to the off-shell enhancement. The use of a fictious sigma-exchange model should not be considered a crucial aspect of the model, since similar forms (summed over a "distribution" of masses m_σ) can be easily obtained also in theoretical approaches based on subtracted dispersion relations. Approaches based on subtracted dispersion relations such as the Bonn πN model [15] lead indeed to similar off-shell enhancement.

For λ_I the extrapolation can be accomplished by invoking ρ-meson dominance and the related Riazuddin-Fayyazuddin-Kawarabayashi-Suzuki identity, which implies (on shell, for $\omega_\pi \to m_\pi$)

$$\frac{\lambda_I^{\rm on}}{m_\pi^2} = \frac{f_\rho^2}{8\pi m_\rho^2}, \tag{5}$$

with the corresponding off-shell extrapolation

$$\begin{aligned}\lambda_I(\tilde{q},\tilde{p}) &= \lambda_I^{\rm on} g_I(\tilde{q},\tilde{p})\\ &= \lambda_I^{\rm on} \frac{m_\rho^2}{m_\rho^2-(\tilde{q}-\tilde{p})^2}\left(\frac{\Lambda_\rho^2}{\Lambda_\rho^2-(\tilde{q}-\tilde{p})^2}\right)^2 .\end{aligned} \tag{6}$$

The production matrix-elements in the non-relativistic $3N$ space with such couplings are the following:

$$\begin{aligned}\langle 3N|A^{\rm 1B}|3N,\pi\rangle &= \frac{-if_{\pi NN}}{m_\pi}\boldsymbol{\sigma}_2\tilde{\mathbf{p}}[\boldsymbol{\tau}_2]_1^{z_\pi}\\ &\times \delta(\mathbf{p}'-\mathbf{p}-\frac{6+2\epsilon}{6(2+\epsilon)}\mathbf{P}_\pi)\,\delta(\mathbf{q}'-\mathbf{q}+\frac{1}{3}\mathbf{P}_\pi)\end{aligned} \tag{7}$$

for the one-body process,

$$\begin{aligned}\langle 3N|A_O^{\rm 2B}|3N,\pi\rangle &= 2i\,\frac{f_{\pi NN}4\pi\lambda_O}{m_\pi^2}\\ &\times \boldsymbol{\sigma}_3\tilde{\mathbf{q}}[\boldsymbol{\tau}_3]_1^{z_\pi}\frac{v_{\pi NN}(\tilde{q})g_O(\tilde{q},\tilde{p})}{m_\pi^2+\tilde{\mathbf{q}}^2-\tilde{q_0}^2}\,\delta(\mathbf{q}'-\mathbf{q}+\frac{1}{3}\mathbf{P}_\pi)\end{aligned} \tag{8}$$

and

$$\begin{aligned}\langle 3N|A_I^{\rm 2B}|3N,\pi\rangle &= \sqrt{2}i\,\frac{f_{\pi NN}4\pi\lambda_I}{m_\pi^3}\\ &\times\boldsymbol{\sigma}_3\tilde{\mathbf{q}}[\boldsymbol{\tau}_3\times\boldsymbol{\tau}_2]_1^{z_\pi}\frac{v_{\pi NN}(\tilde{q})g_I(\tilde{q},\tilde{p})}{m_\pi^2+\tilde{\mathbf{q}}^2-\tilde{q_0}^2}\,(\tilde{q}_0+\omega_\pi)\\ &\times\delta(\mathbf{q}'-\mathbf{q}+\frac{1}{3}\mathbf{P}_\pi)\end{aligned} \tag{9}$$

for the two-body isoscalar and isovector rescattering, respectively. $v_{\pi NN}(\tilde{q})$ is the hadronic form-factor of the πNN vertex, whose structure is governed by the monopole-type cut-off Λ_π. The momenta $\mathbf{p}$ and $\mathbf{q}$ are Jacobi momenta for nucleon 2 in the (2+3) center-of-mass (c.m.), and nucleon 1 in the (1+2+3) c.m., respectively, while $\mathbf{P}_\pi$ is the pion momentum in the total c.m. Similarly, $\mathbf{p}'$ and $\mathbf{q}'$ are the Jacobi momenta for the three nucleons in the no-pion case. Other relevant pion momenta are $\tilde{\mathbf{p}} \simeq \frac{(3+\epsilon)}{3(1+\epsilon)}\mathbf{P}_\pi$ and $\tilde{\mathbf{q}} \simeq \mathbf{p} + \frac{(6+2\epsilon)}{6(2+\epsilon)}\mathbf{P}_\pi - \mathbf{p}'$. In the actual calculation ranging from threshold up to the Δ resonance the on-shell couplings are further corrected by means of an Heitler-type (or K-matrix) form (η is the pion momentum in pion-mass units

$$\hat{\lambda}_O \simeq \frac{2}{3}\frac{\lambda_O+\lambda_I}{1+2i\eta(\lambda_O+\lambda_I)} + \frac{1}{3}\frac{\lambda_O-2\lambda_I}{1+2i\eta(\lambda_O-2\lambda_I)}\,, \tag{10}$$

$$\hat{\lambda}_I \simeq \frac{1}{3}\frac{\lambda_O+\lambda_I}{1+2i\eta(\lambda_O+\lambda_I)} - \frac{1}{3}\frac{\lambda_O-2\lambda_I}{1+2i\eta(\lambda_O-2\lambda_I)}\,. \tag{11}$$

This reduces the rescattering contributions at higher energies but the correction is uninfluential in the threshold limit. On top of these processes, we have included also the 2-body mechanism mediated by Δ rescattering,

$$\langle 3N|A_{\Delta}^{2\mathrm{B}}|3N,\pi\rangle = \frac{-if_{\pi N\Delta}}{m_\pi} \times \frac{V_{N\Delta}(\mathbf{p}',\mathbf{p}_\Delta)\boldsymbol{S}_2\tilde{\mathbf{p}}[\boldsymbol{T}_2]_1^{z_\pi}}{T_{\mathrm{cm}}+M-\mathcal{M}_\Delta+p_\Delta^2/2\mu_\Delta+{q'}^2/2\nu_\Delta} \times \delta(\mathbf{q}'-\mathbf{q}+\frac{1}{3}\mathbf{P}_\pi), \quad (12)$$

since its contribution becomes soon important as the energy increases. The intermediate Δ momentum is defined as $\mathbf{p}_\Delta \simeq \mathbf{p} + \frac{(6+2\epsilon)}{6(2+\epsilon)}\mathbf{P}_\pi$. In Eq. (12) μ_Δ is the reduced mass of the Δ-N system, while ν_Δ is the reduced mass for the N-(ΔN) partition. T_{cm} is the c.m. kinetic energy of the three nucleons in the initial state. For complete details on the employed transition potential, and for other aspects connected with the isobar mechanism, such as, *e.g.* the detailed expression for the complex Δ mass $\mathcal{M}_\Delta$, see Ref. [16] and references therein. All amplitudes Eqs. (7,8,9,12) must be multiplied in addition by the common factor $1/\sqrt{(2\pi)^3\,2\omega_\pi}$. Moreover, taking into account Pauli identity, the full one-body mechanism results by multiplying Eq. (7) by the multiplicity factor $\sqrt{3}(1+P)$, while the remaining two-body mechanisms are multiplied by $2\sqrt{3}(1+P)$, where P is the ternary permutator which commutes the $3N$ coordinates cyclically/anticyclically. Combining the P operator with the given mechanisms in $3N$ partial waves is not a trivial task, and numerical treatment of the resulting amplitudes has been a challenge.

The matrix elements are calculated between in and out nuclear states, where the out-state is specified by the three-nucleon bound-state wave function (plus a free pion wave), and the incoming state is determined by the deuteron-nucleon asymptotic channel. For the $3N$ bound state we have taken the triton wave function as has been developed, tested and calculated in Ref. [17]. As two-body input for the three-nucleon equations we used a separable representation [18] of the Paris interaction. This form represents an analytic version of the PEST interaction, constructed and applied by the Graz group [19].

We have in addition calculated the relevance of the three-nucleon dynamics in the initial state (ISI) by solving the Alt-Grassberger-Sandhas (AGS) equations [20]. More details can be found in [8,21] and references therein.

To exhibit the relevance of isoscalar off-shell effects for $pd \to \pi^+ t$ we have calculated the integral cross section near threshold up to the Δ region. The calculated results are compared with a collection of data from Refs. [9,22,23] and others as explained in Fig. 1. Practically all the experimental data near threshold refer in fact to π^o production from pd collisions, and the comparison has been made assuming isospin invariance and hence implying a factor of two between the two cross sections. In so doing we have avoided the need to include the effects of Coulomb distortions in the exit channel. Given the complexity of the reaction dynamics which depends upon several contributions, the isoscalar effects have been calculated on top of the other mechanisms we had considered. As explained previously, the model includes also p-wave Δ-rescattering, the one-body p-wave term, and the isovector ρ-exchange mechanisms.

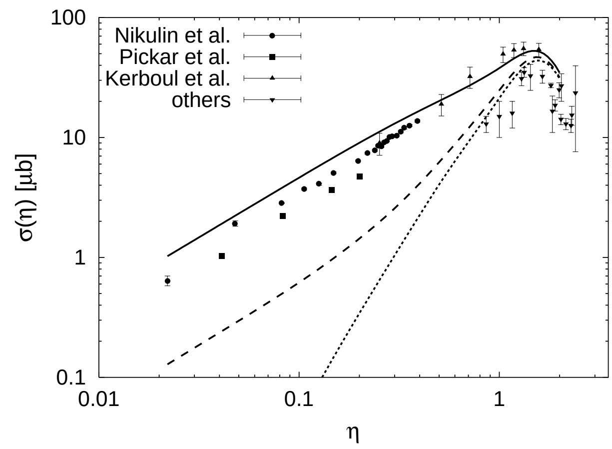

FIG. 1. Production cross section for the $pd \to \pi^+ t$ versus η. The dotted line contains the sole p-wave mechanisms. The dashed line includes also πN s-wave rescattering mediated by ρ-exchange. The solid line considers in addition the isoscalar off-shell effects. The data are from Refs. [9,25,26]. The remaining data ("others") have been extracted from a world collection as explained in Ref. [8].

The relevant parameters employed herein (cut-offs, coupling constants) have been tested previously against the $pp \to \pi^+ d$ process in Ref. [16]. For the ρ-exchange model we use $\lambda_I = 0.045$ in Eq. (5). The results indicate that the modifications introduced by the isoscalar term is significant over the whole range considered, and that

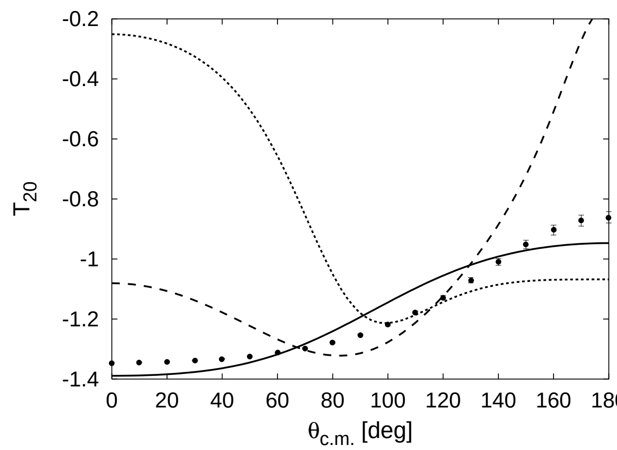

FIG. 2. The deuteron tensor analyzing power T_{20} for $\eta = 0.25$. The lines show the same calculations as in Fig. 1. The points are extracted from Ref. [9].

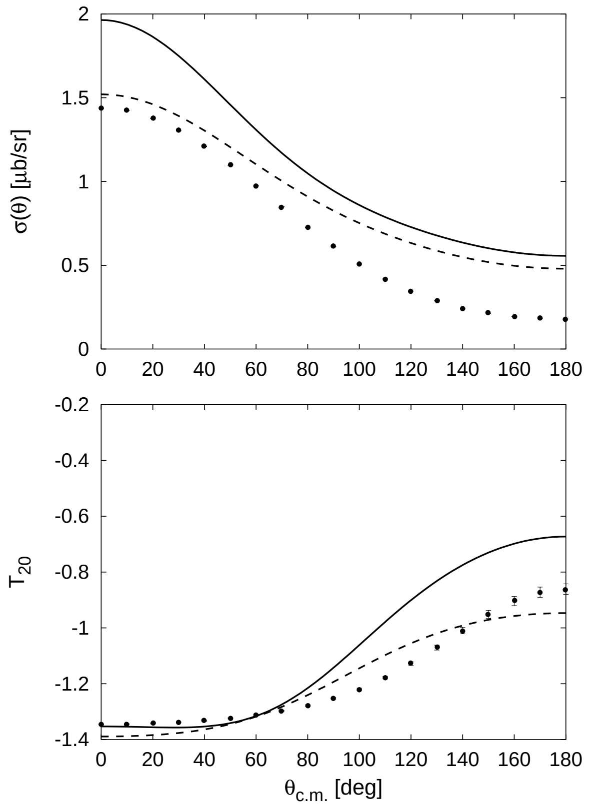

FIG. 3. Effect of $3N$ initial-state correlation at $\eta = 0.25$. Differential cross section (T_{20}) on the top (bottom) panel. In both cases the solid line includes the $3N$ ISI effects, while the dashed line has been obtained in plane-wave approximation. The points are extracted from Ref. [9].

the effect is one of the most pronounced at threshold.

Further evidences come from the results exhibited in Fig. 2, where the deuteron tensor analyzing power T_{20} is shown. Details on the formalism for the calculation of polarization observables can be found in Ref. [24]. The points represent a best-fit to experimental data as given in Ref. [9]. The trend of the data is reproduced once both the isovector *and* isoscalar terms are taken into account.

It is clearly important to assess the role of the initial state correlations, since the three-body dynamics between the nucleon and the deuteron could modify the whole picture and undermine the conclusions of this work. For this reason, we have calculated the ISI effects by solving the AGS equations for the $3N$ system using as input a separable representation of the Paris interaction. The same Faddeev-like technique herein employed has been applied previously to pion production at the Δ resonance in Ref. [8]. In Fig. 3 one can examine the role of the $3N$ dynamics in the initial state from the angular dependence of the unpolarized production cross section and from T_{20}, for $\eta = 0.25$. The modifications introduced by the $3N$ dynamics are sizable, but the overall picture does not change drastically. In addition, the $3N$ effects improve the angular dependence of both observables, thus possibly confirming our findings about the importance of the isoscalar off-shell effects.

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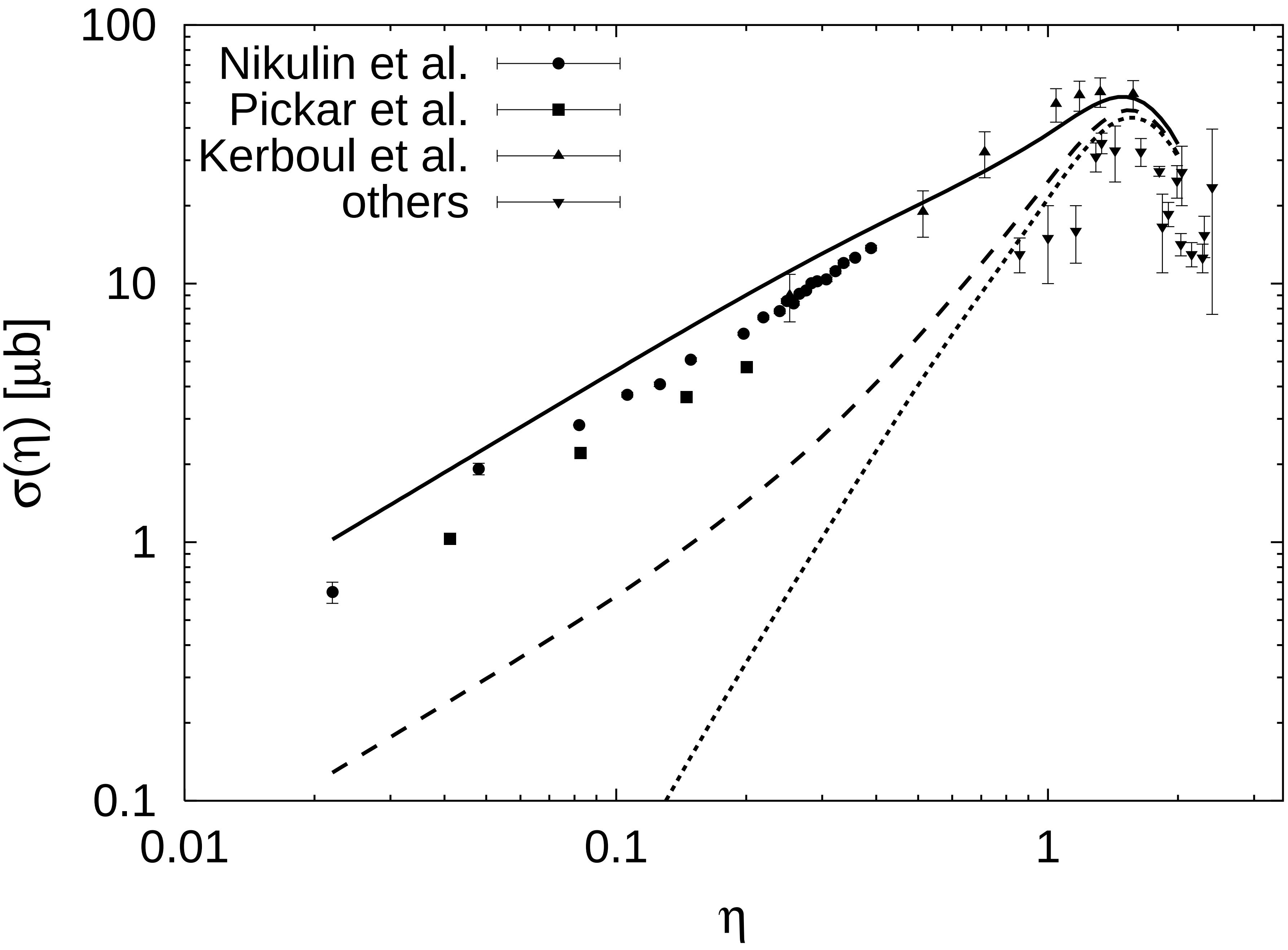
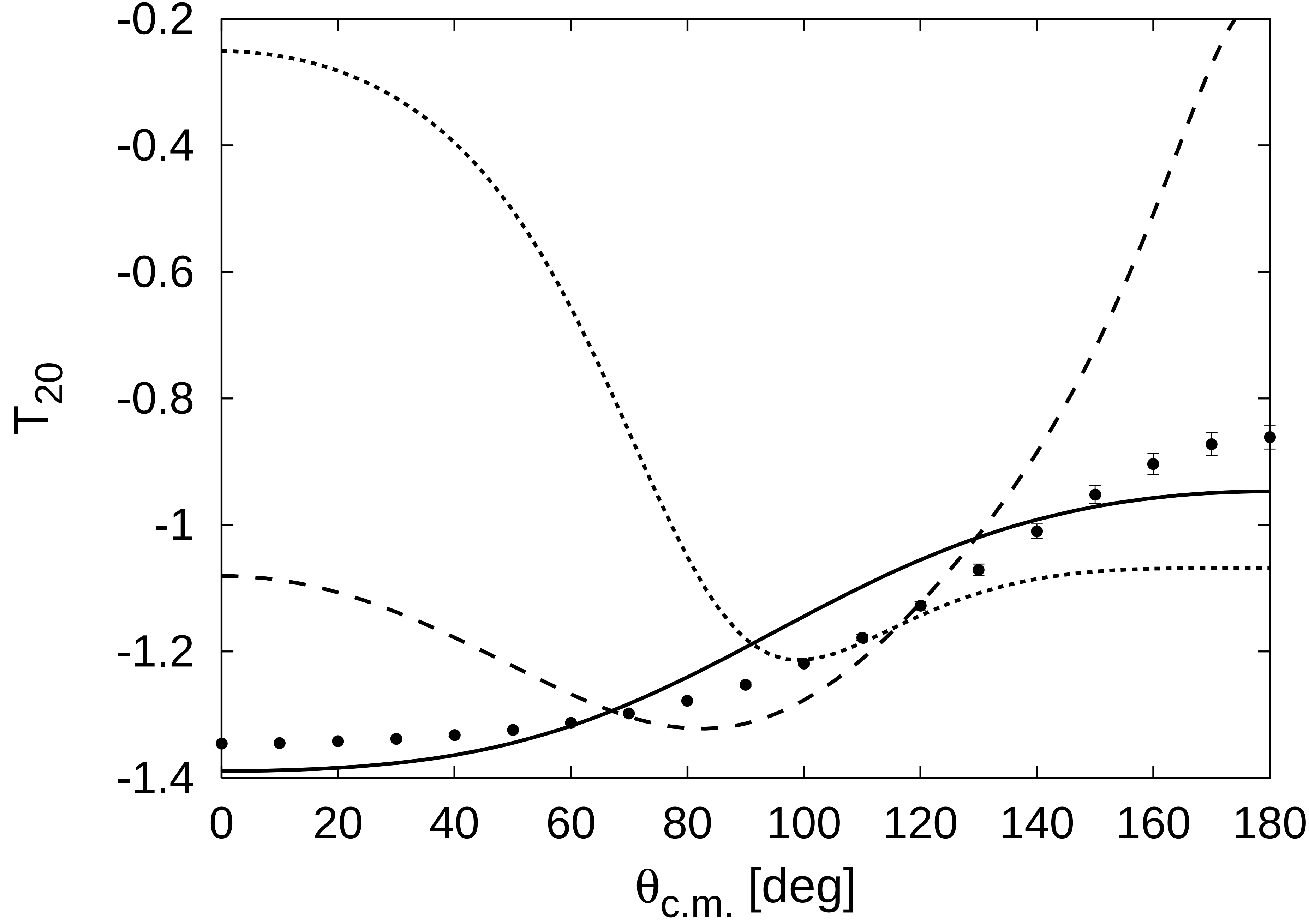
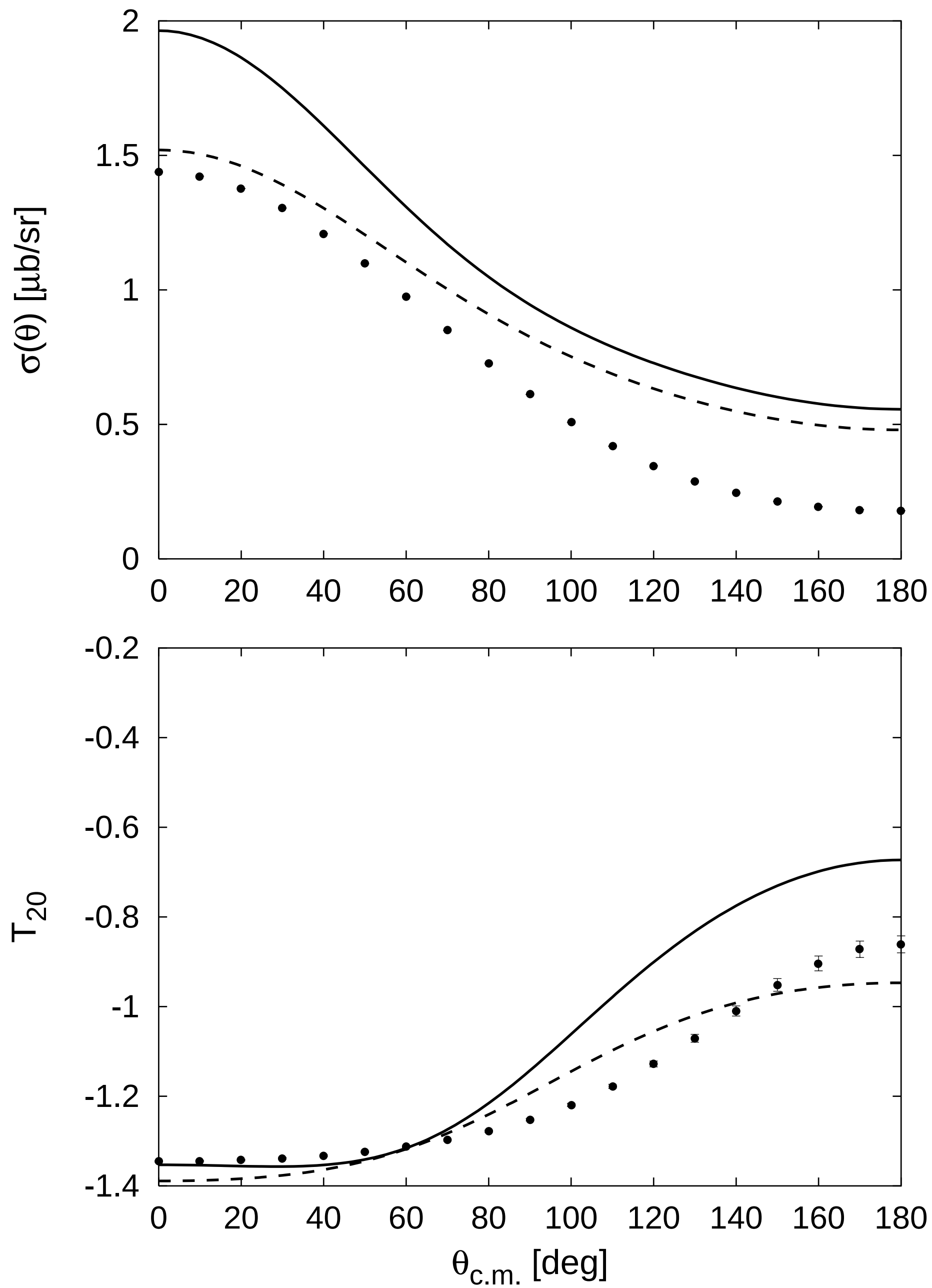

Nikulin et al.
Pickar et al.
Kerboul et al.
others
100
10
1
0.1
0.01
0.1
1
σ(η) [μb]
η



-0.2
-0.4
-0.6
-0.8
-1
-1.2
-1.4
T_{20}
0
20
40
60
80
100
120
140
160
180
$\theta_{c.m.}$ [deg]



σ(θ) [μb/sr]
2
1.5
1
0.5
0
0 20 40 60 80 100 120 140 160 180
T20
-0.2
-0.4
-0.6
-0.8
-1
-1.2
-1.4
0 20 40 60 80 100 120 140 160 180
θc.m. [deg]